ELECTRICAL MODELING OF MAIN INJECTOR QUADRUPOLE MAGNETS

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1. Introduction

The electrical models for three different kinds of quadrupole magnets (116" quad, 100"quad, and 84" quad) are obtained based on three terminal device impedance matrix measurement. The measurement data are analyzed and curve fitted into their equivalent circuits by using circuit simulation program Spice.

2. Electrical Measurement

The quadrupole magnet is a three-terminal device. Fig 1 depicts the quadrupole magnet three-terminal representation. Terminal 1 and 2 are the coil bus terminals and terminal 3 is the magnet case ground. The electrical characteristics of the magnet at non saturation can be described by its admittance matrix. The equations for this three-terminal device network can be written as

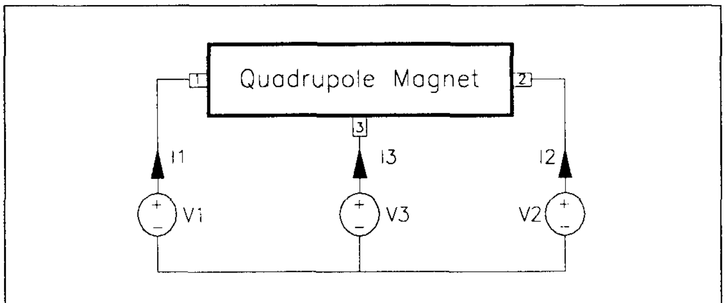

Fig 1 Quadrupole Three-Terminal Representation

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad (1)$$

The 3×3 matrix on the right hand side of Eqn. (1) is called shorted circuit admittance matrix of the considered three-terminal quadrupole magnet. The elements in the shorted circuit admittance matrix are frequency dependent variables. The way to measure this 3×3 matrix elements is depicts in figure 2. The excitation source used here is a high power frequency generator (Elgar Model 500) that can output current up to 5 amps. the output voltage can be adjusted from 0 to 150 Vrms, and the frequency can be varied from 10 Hz to 10KHz. A Tektronix current probe, which has the bandwidth of DC to 50 MHz, was used for current measurement. Both voltage and current as well as phase shifted between voltage and current were measured by Tektronix scope.

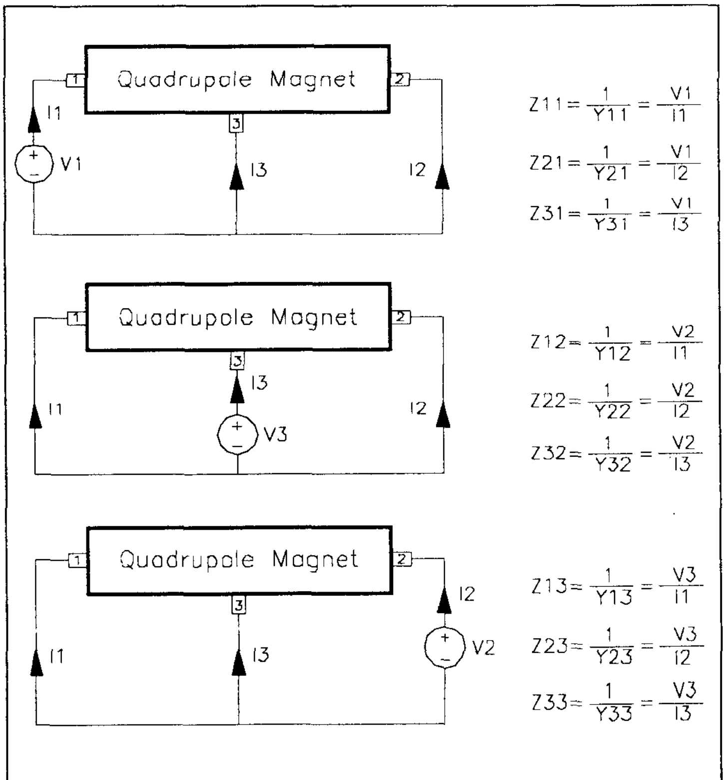

Fig 2 Admittance Matrix Measurement

Ratio of peak to peak value of current to voltage was obtained for the shorted circuit admittance at the frequency of interest. Scope measurement for voltage and current made sure the signals being measured were not distorted.

3. Measurement Data Fitting

DC Resistance of the coil bus is measured at 18 °C. The resistance then is scaled to the resistance at 40 °C by using the formula:

$$R(40^o C) = R(18^o C) \cdot [1 + 0.004 /^o C \cdot (40^o C - 18^o C)] \qquad (2)$$

Coil Impedance measurement is performed by Z_{11}, Z_{22}, Z_{12}, Z_{21}. Z_{11} and Z_{22} are equal both in magnitude and phase for the frequency up to 10 KHz. Z_{12} and Z_{21} are also the same both in magnitude and phase. Z_{11} and Z_{12} are the same in magnitude but 180° out of phase because the reverse direction of the current. This implies the Quadrupole is symmetrical since impedance looking into both coil bus terminals is equal.

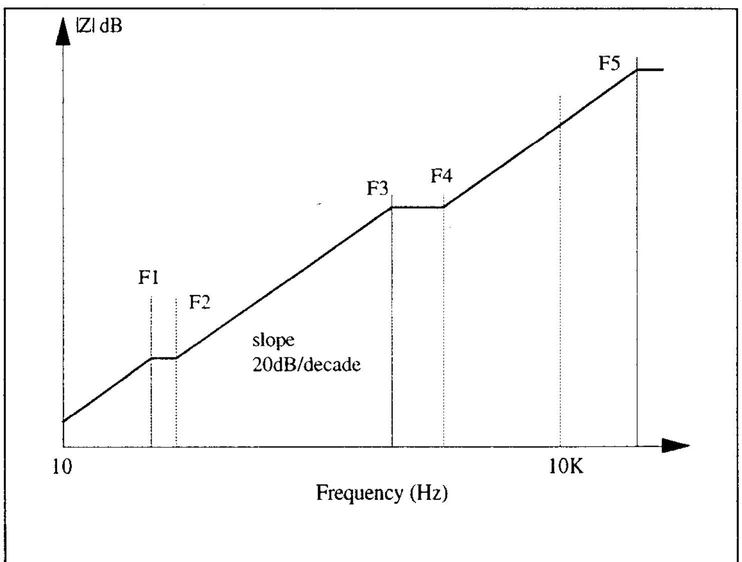

Fig 3 Coil Impedance Data Asymptote Representation

By analyzing the coil impedance measurement data Z_{11} and Z_{22}. The data can be represented by straight-line asymptotes as shown in Fig 3. The bus DC resistance has effect on coil impedance at very low frequency(<< 10 Hz), therefore it is not shown in the impedance asymptote representation here. The coil bus impedance is inductive at the frequency between 10 Hz and F1, F2 and F3, F4 and F5 because the slope of the magnitude asymptote line is 20 dB/decade and the phase is 90°. The coil bus becomes small resistive at the frequency between *F1* and *F2*, *F3* and *F4*. and above *F5*. In the other word, the inductance of the coil decreases as the frequency increases.

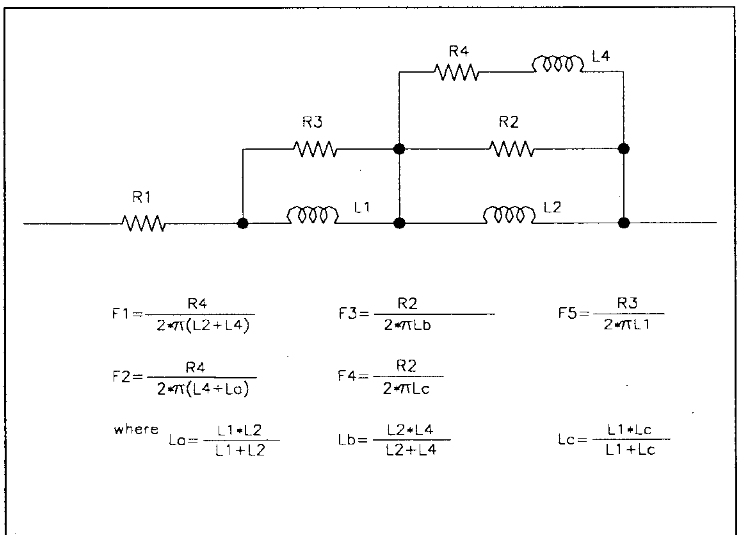

Fig 4 Circuit For Fig 3 Straight-Line Asymptote

An electrical circuit shown in Fig 4 can be used to represent the straight-line asymptote characteristics in Fig 3. The corner frequency break points are approximately given by the formula in figure 4.

Bus Capacitance Measurement is obtained by Z_{13}, Z_{31}, Z_{23}, Z_{32} and Z_{33}. Z_{33} measures the total bus to ground capacitance. Z_{13} and Z_{31} measure the capacitance between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{23} and Z_{32} measure the capacitance between terminal 2 and ground while terminal 1 is shorted. The Z_{13}, Z_{31}, Z_{23}, Z_{32}, and Z_{33} are capacitance measurement because the slope of the measurement data is -20dB/decade in the magnitude plot. The capacitance is determined by the following formula

$$C = \frac{1}{2 \cdot \pi \cdot f \cdot |Z|} \qquad (3)$$

where f is the excitation frequency in Hz and Z is the impedance in ohms.

4. 116" Quad Electrical Model

The impedance matrix measurement data for the 116" Quad are given in Appendix A. The DC resistance is measured at 18 °C and the value is 5.96 mΩ. The resistance at 40 °C is calculated to be 6.48 mΩ by using Eqn (2). R_1 in Fig 4 is the DC resistance.

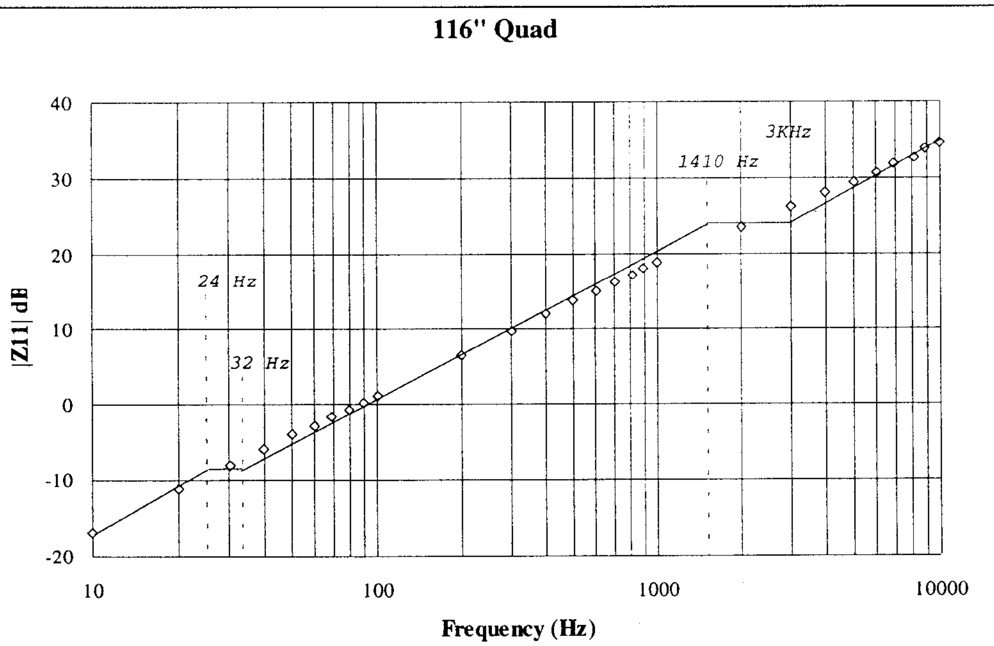

Fig 5 Straight Line Asymptote Representation of 116" Quad Coil Impedance

Coil Impedance measurement is done by Z_{11}, Z_{22}, Z_{12}, Z_{21}. They all have the same magnitude as function of frequency. Figure 5 shows Z_{11} measurement data where the data can be represented by straight line asymptotes. The parameters in the circuit (Fig 4) can be determined according to the data. Referring to Fig 5, the corner frequency break points, the total coil inductance at low frequency (10 Hz), the inductance at high frequency (10KHz) are obtained as:

$F1$=24 Hz $\quad$ $F2$=32 Hz $\quad$ $F3$=1410 Hz $\quad$ $F4$=3000 Hz

Total Inductance @ 10 Hz: $L_T = \frac{1}{2\pi f} \log^{-1}[\frac{|Z_{11}|}{20}] = 2.2$ mH where $|Z_{11}| = -17.2$ dB

Inductance @ 10 KHz: $L_1 = \frac{1}{2\pi f} \log^{-1}[\frac{|Z_{11}|}{20}] = 0.8$ mH where $|Z_{11}| = 34$ dB

The value of the circuit elements can be calculated since the total coil inductance, inductance at high frequency, and the corner frequency break points are known.

L2 Inductance: $L_2 = L_T - L_1 = 1.4$ mH

L4 Inductance: $$L_4 = \frac{L_1 \cdot L_2 \cdot (F4 - F3)}{(L_1 + L_2) \cdot F3 - L_1 \cdot F_4} = 2.5 \text{ mH}$$

R2: $$R_2 = 2 \cdot \pi \cdot F3 \cdot \frac{L_2 \cdot L_4}{L_2 + L_4} = 8\ \Omega$$

R4: $$R_4 = 2 \cdot \pi \cdot F1 \cdot (L_2 + L_4) = 0.6\ \Omega$$

Resistor R3 is added to match the coil impedance phase at high frequency although we don't see any significant changes in magnitude plot since the pole introduced by R_3 and L_1 is located at 50 KHz. The value of R_3 is 250 Ω.

Bus Capacitance is measured by Z_{13}, Z_{31}, Z_{23}, Z_{32}, and Z_{33}. The total bus to magnet case ground capacitance (C_T) is measured by Z_{33}. Z_{13} and Z_{31} measure the capacitance (C_1) between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{23} and Z_{32} measure the capacitance (C_2) between terminal 2 and ground while terminal 1 is grounded.

Total Bus to Ground Capacitance: $$C_T = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{55}|} = 54 \text{ nF}$$

Terminal 1 to Ground Capacitance: $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{15}|} = 27 \text{ nF}$$ or $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{51}|} = 27 \text{ nF}$$

Terminal 2 to Ground Capacitance: $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{25}|} = 27 \text{ nF}$$ or $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{52}|} = 27 \text{ nF}$$

The 116" quadrupole magnet electrical model is shown in Fig 6. The simulation results are given in Appendix A. The model matches the measurement data up to 10 Khz.

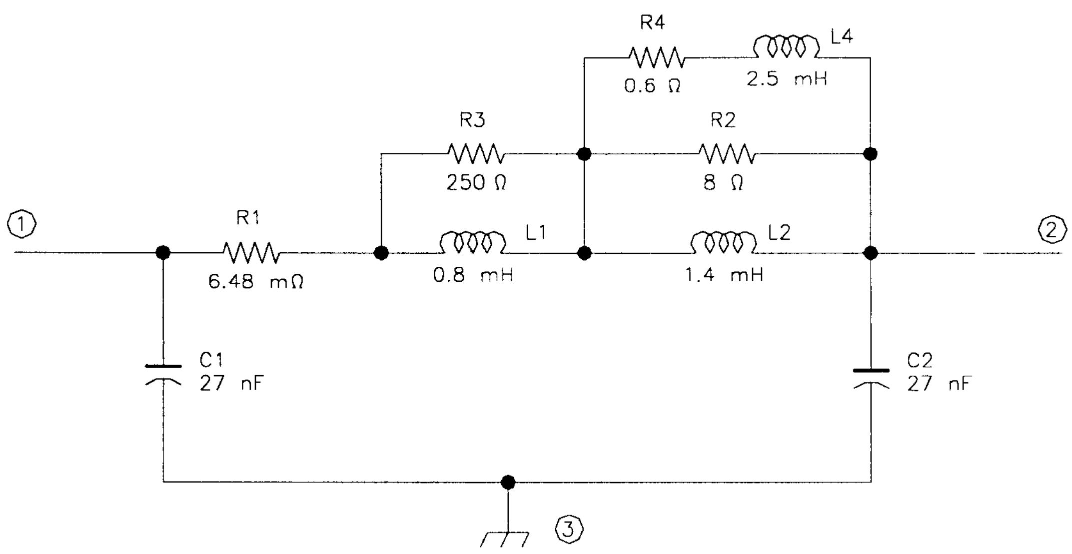

Fig 6 Main Injector 116" Quad Electrical Model

5. 100" Quad Electrical Model

The impedance matrix measurement data for the 100" Quad are given in Appendix B. The DC resistance is measured at 18 °C and the value is 4.84 mΩ. The resistance at 40 °C is calculated to be 5.27 mΩ by using Eqn (2). R_1 in Fig 4 is the DC resistance.

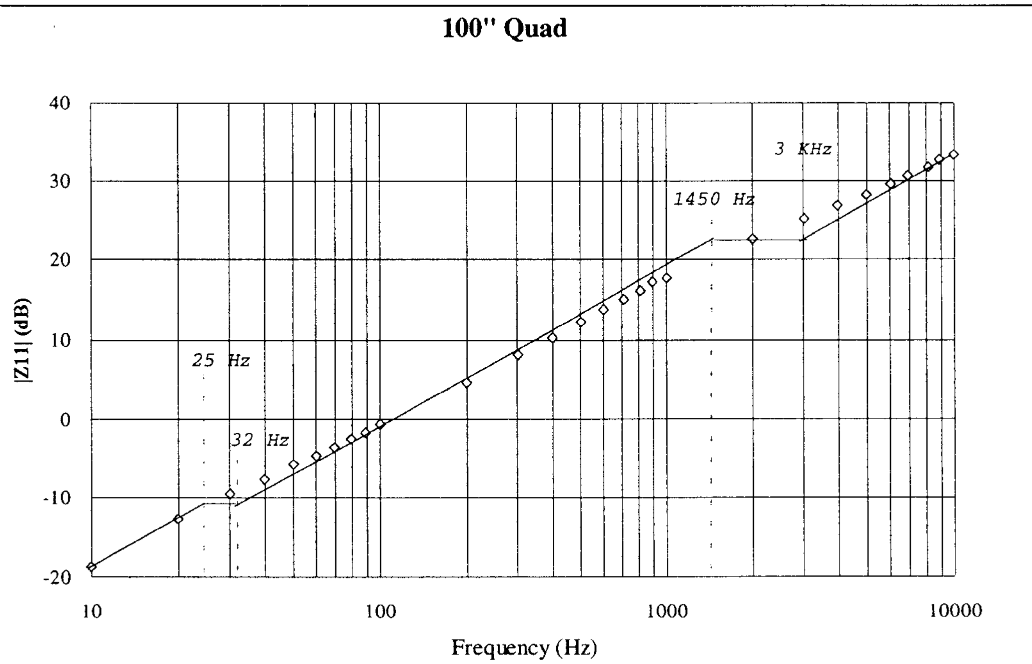

Fig 7 Straight Line Asymptote Representation of 100" Quad Coil Impedance

Coil Impedance measurement is performed by Z_{11}, Z_{22}, Z_{12}, Z_{21}. They all have the same magnitude as function of frequency. Figure 7 shows Z_{11} measurement data where the data can be represented by straight line asymptotes. The parameters in the circuit (Fig 4) can be determined according to the data. Referring to Fig 7, the corner frequency break points, the total coil inductance at low frequency (10 Hz), the inductance at high frequency (10KHz) are obtained as:

$F1$=25 Hz $F2$=32 Hz $F3$=1450 Hz $F4$=3000 Hz

Total Inductance @ 10 Hz: $L_T = \frac{1}{2\pi f}\log^{-1}[\frac{|Z_{11}|}{20}] = 1.9$ mH where $|Z_{11}| = -18.5$ dB

Inductance @ 10 KHz: $L_1 = \frac{1}{2\pi f}\log^{-1}[\frac{|Z_{11}|}{20}] = 0.7$ mH where $|Z_{11}| = 32.5$ dB

The value of the circuit elements can be calculated since the total coil inductance, inductance at high frequency, and the corner frequency break points are known.

L2 Inductance: $L_2 = L_T - L_1 = 1.2$ mH

L4 Inductance: $$L_4 = \frac{L_1 \cdot L_2 \cdot (F4 - F3)}{(L_1 + L_2) \cdot F3 - L_1 \cdot F4} = 2 \text{ mH}$$

R2: $$R_2 = 2 \cdot \pi \cdot F3 \cdot \frac{L_2 \cdot L_4}{L_2 + L_4} = 7\ \Omega$$

R4: $$R_4 = 2 \cdot \pi \cdot F1 \cdot (L_2 + L_4) = 0.5\ \Omega$$

Resistor R3 is added to match the coil impedance phase at high frequency although we don't see any significant changes in magnitude plot since the the pole introduced by R_3 and L_1 is located at 50 KHz. The value of R_3 is 220 Ω.

Bus Capacitance is measured by Z_{15}, Z_{51}, Z_{25}, Z_{52}, and Z_{55}. The total bus to magnet case ground capacitance (C_T) is measured by Z_{55}. Z_{15} and Z_{51} measures the capacitance (C_1) between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{25} and Z_{52} measures the capacitance (C_2) between terminal 2 and ground while terminal 1 is grounded.

Total Bus to Ground Capacitance: $$C_T = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{55}|} = 48 \text{ nF}$$

Terminal 1 to Ground Capacitance: $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{15}|} = 24 \text{ nF}$$ or $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{51}|} = 24 \text{ nF}$$

Terminal 2 to Ground Capacitance: $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{25}|} = 24 \text{ nF}$$ or $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{52}|} = 24 \text{ nF}$$

The 100" quadrupole magnet electrical model is shown in Fig 8 and the simulation results are given in Appendix B. The model matches the measurement data up to 10 KHz.

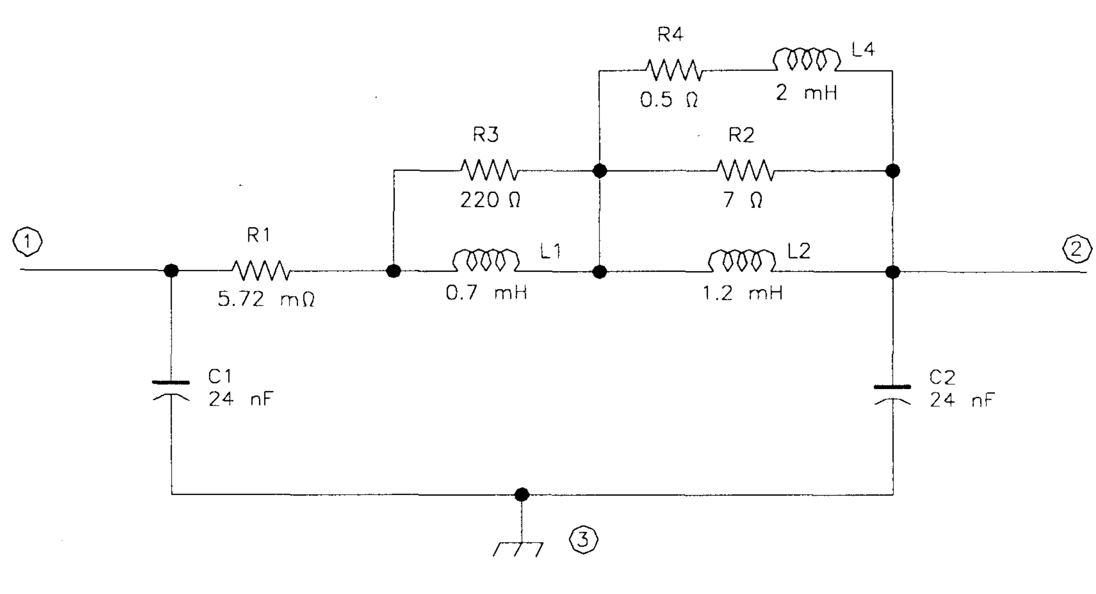

Fig 8 Main Injector 100" Quad Electrical Model

6. 84" Quad Electrical Model

The impedance matrix measurement data for the 84" Quad are given in Appendix C. The DC resistance is measured at 18 °C and the value is 4.44 mΩ. The resistance at 40 °C is calculated to be 4.83 mΩ by using Eqn (2). R_1 in Fig 4 is the DC resistance.

Coil Impedance measurement is obtained by Z_{11}, Z_{22}, Z_{12}, Z_{21}. They all have the same magnitude as function of frequency. Figure 9 shows Z_{11} measurement data where the data can be represented by straight line asymptotes. The parameters in the circuit (Fig 4) can be determined according to the data. Referring to Fig 9, the corner frequency break points, the total coil inductance at low frequency (10 Hz), the inductance at high frequency (10KHz) are obtained as:

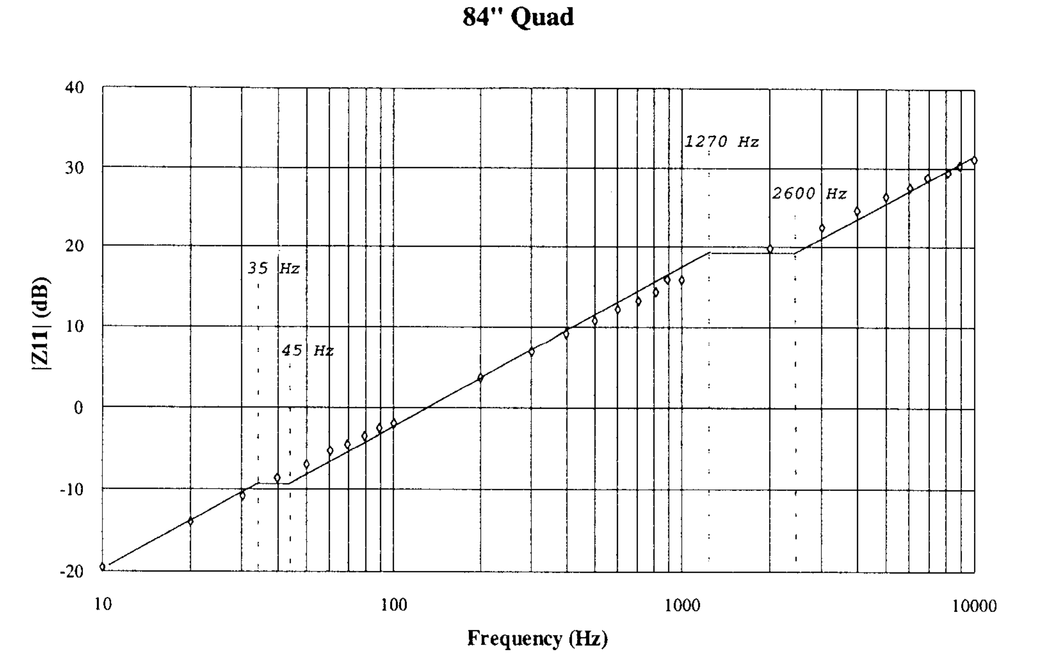

Fig 9 Straight Line Asymptote Representation of 116" Quad Coil Impedance

$F1$=35 Hz $F2$=45 Hz $F3$=1270 Hz $F4$=2600 Hz

Total Inductance @ 10 Hz: $L_T = \frac{1}{2\pi f} \log^{-1}[\frac{|Z_{11}|}{20}] = 1.6$ mH where $|Z_{11}|$ = - 20 dB

Inductance @ 10 KHz: $L_1 = \frac{1}{2\pi f} \log^{-1}[\frac{|Z_{11}|}{20}] = 0.6$ mH where $|Z_{11}|$ = 31.5 dB

The value of the circuit elements can be calculated since the total coil inductance, inductance at high frequency, and the corner frequency break points are known.

L2 Inductance: $L_2 = L_T - L_1 = 1$ mH

L4 Inductance: $L_4 = \frac{L_1 \cdot L_2 \cdot (F4 - F3)}{(L_1 + L_2) \cdot F3 - L_1 \cdot F4} = 1.7$ mH

R2: $R_2 = 2 \cdot \pi \cdot F3 \cdot \frac{L_2 \cdot L_4}{L_2 + L_4} = 5\ \Omega$

R4: $R_4 = 2 \cdot \pi \cdot F1 \cdot (L_2 + L_4) = 0.6\ \Omega$

Resistor R3 is added to match the coil impedance phase at high frequency although we don't see any significant changes in magnitude plot since the pole introduced by R_3 and L_1 is located at 49 KHz. The value of R3 is 185 Ω.

Bus Capacitance is measured by Z_{13}, Z_{31}, Z_{23}, Z_{32}, and Z_{33}. The total bus to magnet case ground capacitance (C_T) is measured by Z_{33}. Z_{13} and Z_{31} measures the capacitance (C_1) between terminal 1 and ground while terminal 2 is shorted to ground. Similarly, Z_{23} and Z_{32} measures the capacitance (C_2) between terminal 2 and ground while terminal 1 is grounded.

Total Bus to Ground Capacitance: $$C_T = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{55}|} = 40 \text{ nF}$$

Terminal 1 to Ground Capacitance: $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{15}|} = 20 \text{ nF}$$ or $$C_1 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{51}|} = 20 \text{ nF}$$

Terminal 2 to Ground Capacitance: $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{25}|} = 20 \text{ nF}$$ or $$C_2 = \frac{1}{2 \cdot \pi \cdot f \cdot |Z_{52}|} = 20 \text{ nF}$$

The 84" quadrupole magnet electrical model is shown in Fig 11. The simulation results are given in Appendix C. The model matches the measurement data up to 10 KHz.

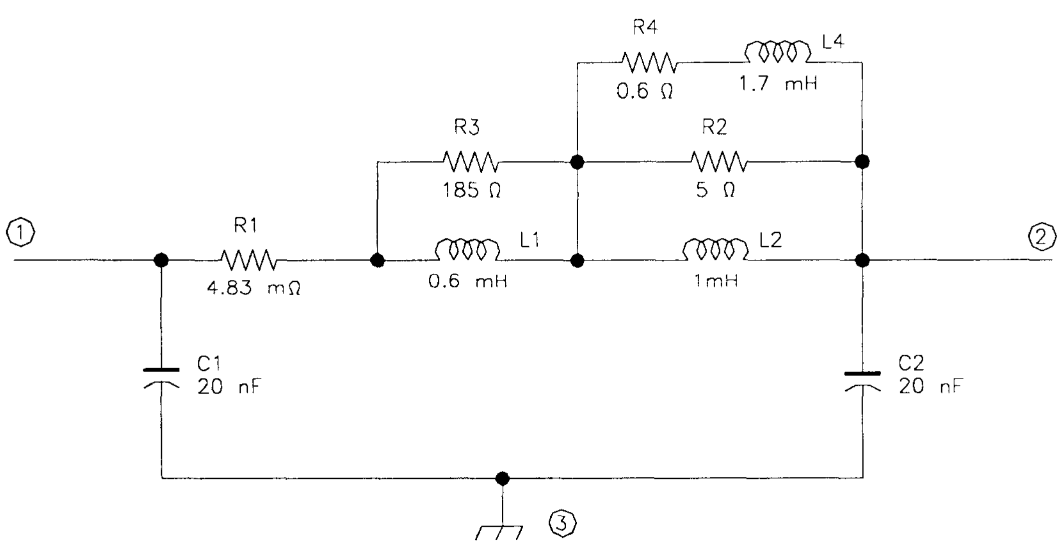

Fig 10 Main Injector 84" Quad Electrical Model

7. Conclusion

The electrical models for 116", 100", and 84" quadrupole magnets are obtained bsed on impedance matrix measurement. Spice simulation result shows the accuracy of the models. The electrical models can be used as a sub circuit to build a quadrupole ring model to study the transient and frequency response of the system.